

# NIMS UNIVERSITY, JAIPUR



## SYLLABUS

**M. Sc. MATHS PREVIOUS**

**M.Sc. (Previous) Mathematics****YEAR - I**

<b>S.N O</b>	<b>PAPER</b>	<b>THEORY</b>	<b>TOTAL</b>
1	Advanced Abstract Algebra	100	100
2	Real Analysis and Topology	100	100
3	Differential Equations	100	100
4	Differential Geometry	100	100
5	Mechanics	100	100

## **Paper - I: Advanced Abstract Algebra**

### **Unit 1:**

Direct product of groups (External and Internal). Isomorphism theorems - Diamond isomorphism theorem, Butterfly Lemma, Conjugate classes (Excluding p-groups), Commutators, Derived subgroups, Normal series and Solvable groups, Composition series, Refinement theorem and Jordan-Holder theorem for infinite groups.

### **Unit 2:**

Euclidean rings. Modules, Submodules, Quotient modules Direct sums and Module Homomorphisms. Generation of modules, Cyclic modules. Linear transformation of vector spaces, Dual spaces, Dual basis and their properties, Dual maps, Annihilator.

### **Unit 3:**

Field theory - Extension fields, Algebraic and Transcendental extensions, Separable and inseparable extensions, Normal extensions. Splitting fields. Galois theory - the elements of Galois theory, Automorphism of extensions, Fundamental theorem of Galois theory, Solutions of polynomial equations by radicals and Insolvability of general equation of degree five by radicals.

### **Unit 4:**

Matrices of a linear maps, Matrices of composition maps, Matrices of dual map, Eigen values, Eigen vectors, Rank and Nullity of linear maps and matrices, Invertible matrices, Similar matrices, Determinants of matrices and its computations, Characteristic polynomial and eigen values.

### **Unit 5:**

Real inner product space, Schwartz inequality, Orthogonality, Bessel's inequality, Adjoint, Self adjoint linear transformations and matrices, Orthogonal linear transformation and matrices, Principal Axis Theorem.

## Paper - II: Real Analysis and Topology

### Unit 1:

Algebra and algebras of sets, Algebras generated by a class of subsets, Borel sets, Lebesgue measure of sets of real numbers, Measurability and Measure of a set, Existence of Non-measurable sets, Measurable functions, Realization of non-negative measurable function as limit of an increasing sequence of simple functions, Structure of measurable functions, Convergence in measure, Egoroff's theorem.

### Unit 2:

Weierstrass's theorem on the approximation of continuous function by polynomials, Lebesgue integral of bounded measurable functions, Lebesgue theorem on the passage to the limit under the integral sign for bounded measurable functions. Summable functions, Space of square summable functions. Fourier series and coefficients, Parseval's identity, Riesz-Fisher Theorem.

### Unit 3:

Lebesgue integration on  $\mathbb{R}^2$ , Fubini's theorem.  $L^p$ -spaces, Holder-Minkowski inequalities. Completeness of  $L^p$ -spaces, Topological spaces, Subspaces, Open sets, Closed sets, Neighbourhood system, Bases and sub-bases.

### Unit 4:

Continuous mapping and Homeomorphism, Nets, Filters, Separation axioms ( $T_0$ ,  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ ). Product and Quotient spaces.

### Unit 5:

Compact and locally compact spaces. Tychonoff's one point compactification. Connected and Locally connected spaces, Continuity and Connectedness and Compactness.

## Paper - III: Differential Equations

### Unit 1:

Non-linear ordinary differential equations of particular forms. Riccati's equation - General solution and the solution when one, two or three particular solutions are known. Total Differential equations. Partial differential equations of second order with variable co-efficients- Monge's method.

### Unit 2:

Classification of linear partial differential equation of second order, Cauchy's problem, Method of separation of variables, Laplace, Wave and diffusion equations, Canonical forms. Linear homogeneous boundary value problems. Eigen values and eigen functions. Strum-Liouville boundary value problems. Orthogonality of eigen functions. Reality of eigen values.

### Unit 3:

Calculus of variation - Functionals, Variation of a functional and its properties, Variational problems with fixed boundaries, Euler's equation, Extremals, Functional dependent on several unknown functions and their first order derivatives, Functionals dependent on higher order derivatives, Functionals dependent on the function of more than one independent variable. Variational problems in parametric form, Series solution of a second order linear differential equation near a regular/singular point (Method of Frobenius) with special reference to Gauss hypergeometric equation and Legendre's equation.

### Unit 4:

Gauss hypergeometric function and its properties, Integral representation, Linear transformation formulas, Contiguous function relations, Differentiation formulae, Linear relation between the solutions of Gauss hypergeometric equation, Kummer's confluent hypergeometric function and its properties, Integral representation, Kummer's first transformation. Legendre polynomials and functions  $P_n(x)$  and  $Q_n(x)$ .

### Unit 5:

Bessel functions  $J_n(x)$ , Hermite polynomials  $H_n(x)$ , Laguerre and Associated Laguerre polynomials.

## Paper - IV: Differential Geometry

### Unit 1:

Theory of curves- Space curves, Tangent, Contact of curve and surface, Osculating plane, Principal normal and Binormal, Curvature, Torsion, Serret-Frenet's formulae, Osculating circle and Osculating sphere, Existence and Uniqueness theorems, Bertrand curves, Involute, Evolutes. Conoids, Inflexional tangents, Singular points, Indicatrix

### Unit 2:

Envelop, Edge of regression, Ruled surface, Developable surface, Tangent plane to a ruled surface. Necessary and sufficient condition that a surface  $\square = f()$  should represent a developable surface. Metric of a surface, First, second and third fundamental forms. Fundamental magnitudes of some important surfaces, Orthogonal trajectories. normal curvature, Meunier's theorem,.

### Unit 3:

Principal directions and Principal curvatures, First curvature, Mean curvature, Gaussian curvature, Umbilics. Radius of curvature of any normal section at an umbilic on  $z = f(x,y)$ . Radius of curvature of a given section through any point on  $z = f(x,y)$ . Lines of curvature, Principal radii, Relation between fundamental forms. Asymptotic lines, Differential equation of an asymptotic line, Curvature and Torsion of an asymptotic line. Geodesics, Differential equation of a geodesic, Single differential equation of a geodesic, Geodesic on a surface of revolution, Geodesic Curvature and Torsion, Gauss-Bonnet Theorem.

### Unit 4:

Gauss's formulae, Gauss's characteristic equation, Weingarten equations, Mainardi-Codazzi equations. Fundamental existence theorem for surfaces, Parallel surfaces, Gaussian and mean curvature for a parallel surface, Bonnet's theorem on parallel surfaces.

Tensor Analysis- Kronecker delta. Contravariant and Covariant tensors, Symmetric tensors, Quotient law of tensors, Relative tensor. Riemannian space. Metric tensor, Indicator, Permutation symbols and Permutation tensors.

### Unit 5:

Christoffel symbols and their properties, Covariant differentiation of tensors. Ricci's theorem, Intrinsic derivative, Geodesics, Differential equation of geodesic, Geodesic coordinates, Field of parallel vectors, Riemann-Christoffel tensor and its properties. Covariant curvature tensor, Einstein space. Bianchi's identity. Einstein tensor, Flat space, Isotropic point, Schur's theorem.

## Paper - V: Mechanics

### Unit 1:

D'Alembert's principle. The general equations of motion of a rigid body. Motion of centre of inertia and motion relative to centre of inertia. Motion about a fixed axis. The compound pendulum, Centre of percussion. Motion of a rigid body in two dimensions under finite and impulsive forces.

### Unit 2:

Motion in three dimensions with reference to Euler's dynamical and geometrical equations. Motion under no forces, Motion under impulsive forces. Conservation of momentum (linear and angular) and energy for finite as well as impulsive forces.

### Unit 3:

Lagrange's equations for holonomous dynamical system, Energy equation for conservative field, Small oscillations, Motion under impulsive forces. Motion of a top. Hamilton's equations of motion, Conservation of energy, Hamilton's principle and principle of least action.

### Unit 4:

Kinematics of ideal fluid. Lagrange's and Euler's methods. Equation of continuity in Cartesian, cylindrical and spherical polar coordinates. Boundary surface. Stream-lines, path-lines and stream lines velocity potential irrotational motion.

### Unit 5:

Euler's hydrodynamic equations. Bernoulli's theorem. Helmholtz equations. Cauchy's integral, Motion due to impulsive forces. Motion in two-dimensions, Stream function, Complex potential. Sources, Sinks, Doublets, Images in two dimensions.